Entropic Image Segmentation: A Fuzzy Approach Based on Tsallis Entropy

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Abstract

In this paper, a fuzzy approach for image segmentation based on Tsallis entropy is introduced. This approach employs fuzzy Tsallis entropy to measure the structural information of image and to locate the optimal threshold desired by segmentation. The proposed method draws upon the postulation that the optimal threshold concurs with maximum information content of the distribution. The contributions of the work are as follow: Initially, fuzzy Tsallis entropy as a measure of spatial structure of image is described. Then, an unsupervised entropic segmentation method based on fuzzy Tsallis entropy is developed. Although the proposed approach belongs to entropic segmentation approaches (i.e., such approaches are commonly applied to grayscale images), it is adapted to be viable for segmenting color images. Finally, substantial experiments are carried out on realistic images to validate the effectiveness, efficiency and robustness of the proposed method.

Keywords: Tsallis entropy, entropic image segmentation, thresholding, fuzzy entropy

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1. Introduction

In computer vision, image segmentation is the process of partitioning a digital image into multiple segments (sets of pixels, also known as superpixels). This is typically used to identify objects or other relevant information in digital images. This process is an elementary and significant component in many applications, such as image analysis, pattern recognition [1, 2], medical diagnosis and currently in robotic vision. However, it is one of the most difficult and challenging tasks in image processing. In the literature, there are several algorithms (using different approaches) for image segmentation proposed by various researchers. Examples of these approaches include local edge detection (e.g. [3]), deformable curves (e.g. [4]), morphological region-based approaches (e.g. [5, 6, 7], global optimization approaches on energy functions and stochastic model-based methods (e.g. [8, 9, 10]).

Recent developments of statistical mechanics based on a concept of nonextensive entropy have intensified the interest of investigating a possible extension of Shannon entropy to Information Theory [11]. This interest appears mainly due to similarities between Shannon and Boltzmann/Gibbs entropy functions. The nonextensive entropy is a new proposal in order to generalize the Boltzmann/Gibbs's traditional entropy to nonextensive systems (i.e. strong correlated systems are good candidates to be nonextensive). In this theory a new parameter q is introduced as a real number associated with the nonextensivity of the system.

In this paper, a new fuzzy method for image segmentation is introduced, which employs for the first time the fuzzy concept on the Tsallis entropy of generalized distributions. The experimental results suggest that the fuzzy Tsallis entropy provides the proposed method competitive segmentation performance in comparison to other existing segmentation methods [12] and they also depict the validity of the proposed method.

The remainder of the paper is structured as follows:

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In Section 2, Tsallis entropy of generalized distributions and nonextensive systems are explained. In Section 3, the proposed method for image segmentation is described in detail. Section 4 describes the experiments performed and presents the results of the experiments. Finally, in Section 5, the main contributions of the paper are outlined and some conclusions are drawn.

2. Tsallis entropy of generalized distributions

Entropy has first appeared in thermodynamics as an information theoretical concept which is intimately related to the internal energy of the system. Then it has applied across physics, information theory, mathematics and other branches of science and engineering [13]. When given a system whose exact description is not precisely known, the entropy is defined as the expected amount of information needed to exactly specify the state of the system, given what we know about the system.

In 1948, the entropy of Boltzmann/Gibbs has been redefined by Shannon [14] as a measure of uncertainty regarding the information content of a system. Therefore, An expression could be defined for measuring the amount of information produced by a process. Formally speaking, let $P = (p_1, p_2, \ldots, p_n)$ be a finite discrete probability distribution, that is, suppose $p_k \geq 0, k = 1, 2, \ldots, n$ and $\sum_{k=1}^{n} p_k = 1$.

The amount of uncertainty of the distribution P (i.e. the amount of uncertainty concerning the outcome of an experiment) is called the entropy of the distribution and is usually measured by the quantity $H(P) = H(p_1, p_2, ..., p_n)$. introduced by Shannon and is given by

$$H(P) = -\sum_{k=1}^{n} p_k \log_2 p_k \tag{1}$$

It is straightforward to demonstrate that the Shannon entropy for the conjunction of two distributions P and Q satisfies the following property:

$$H(P + Q) = H(P) + H(Q) \tag{2}$$

The above formula provides one of the most important properties of entropy, namely, its additivity; the entropy of a combined experiment consisting of the performance of two independent experiments is equal to the sum of the entropies of these two experiments. The formalism defined by Eq. (1) has been shown to be restricted to the Boltzmann-Gibbs-Shannon (BGS) statistics. However, for nonextensive systems, some kind of extension appears to become necessary.

In 1988, C. Tsallis [15] proposed a wider class of entropies as a basis for generalizing the standard statistical mechanics. Such an entropy is a generalization of the standard Boltzmann-Gibbs which is useful for describing the properties of nonextensive systems entropy and defined as,

$$H_q(P) = \frac{1}{q-1}\left[1 - \sum_{k=1}^{n} p_k^q\right] \tag{3}$$

where $q \geq 0$ and $q \neq 1$. The real number q is called an entropic order that characterizes the degree of nonextensivity. This expression reduces to Shannon entropy in the limit $q \longrightarrow 1$. Thus, Shannon's measure of entropy is the limiting of the measure of entropy H_q and it is called the measure of entropy of order 1 of the distribution.

It is worth mentioning that the parameter q in Tsallis entropy is typically interpreted as a quantity characterizing the degree of nonextensivity of a physical system [16]. In some cases the parameter q has no physical meaning, but it gives new possibilities in the agreement of theoretical models and experimental data [17]. In other cases, q is solely determined by constraints of the problem and by this means q may have a physical meaning [18].

We shall see that in order to get the fine characterization of Tsallis entropy, it is advantageous to extend the notion of a probability distribution, and define entropy for the generalized distributions. The characterization of measures of entropy (and information) becomes much simpler if we consider these quantities as defined on the set of generalized probability distributions.

Let $[\Omega, P]$ be a probability space that is, Ω an arbitrary nonempty set, called the set of elementary events, and P a probability measure, that is, a nonnegative and additive set function for which $P(\Omega_1) = 1$. Let us call a function $\xi = \xi(\omega)$ which is defined for $\omega \in \Omega_1$, where $\Omega_1 \subset \Omega$. If $P(\Omega_1) = 1$ we call ξ an ordinary (or complete) random variable, while if $0 < P(\Omega_1) \leq 1$ we call ξ an incomplete random variable. Evidently, an incomplete random variable can be interpreted as a quantity describing the result of an experiment depending on chance which is not always observable, only with probability $P(\Omega_1) < 1$. The distribution of a generalized random variable is called a generalized probability distribution. Thus a finite discrete generalized probability distribution is simply a sequence $p_1, p_2, ..., p_n$ of nonnegative numbers such that setting $P = \{p_k\}_{k=1}^{n}$ and taking,

$$\varpi(P) = \sum_{k=1}^{n} p_k \tag{4}$$

where $\varpi(P)$ is the weight of the distribution and $0 < \varpi(P) \leq 1$. A distribution that has a weight less than 1 will be called an incomplete distribution. Now, using equations (3) and (4), Tsallis entropy defined on the generalized distribution can be written as follows,

$$H_q(P) = \frac{1}{q-1}\left[1 - \frac{\sum_{k=1}^{n} p_k^q}{\sum_{k=1}^{n} p_k}\right] \tag{5}$$

Note that Tsallis entropy has a nonextensive property for statistical independent systems. This property is defined by the following pseudo additivity entropic formula:

$$H_q(A+B) = H_q(A) + H_q(B) + (1-q) \cdot H_q(A) \cdot H_q(B) \tag{6}$$

3. Suggested Methodology

The study of the experimental results of several segmentation techniques has shown in the occurrence of too much noise in the image, the process of segmentation becomes a tricky. While there are much more techniques for image segmentation, some of them are time-consuming and the others call for huge storage space. The proposed technique achieves the task of segmentation in a novel way. This technique not only surmounts the noise in image but also it calls for neither more time nor massive storage space. This happens by the advantage of using fuzzy Tsallis entropy of generalized distributions to measure the structural information of image and then locate the optimal threshold depending on the postulation that the optimal threshold corresponds to the segmentation with maximum structure (i.e., maximum information content of the distribution). The proposed technique methodologically comprises the following main steps:

3.1. Preprocessing

In order to suppress of image noise, we employ Gaussian smoothing to convolve the input image with a Gaussian operator. Additionally, in this step, we apply a dedicated filter to get rid of small isolated noisy points [19, 1].

3.2. Fuzzy entropic thresholding

This step takes in the subsequent sub steps:

3.2.1. Entropy calculation

For an input image, let $\{p_i\}_{i=1}^{n}$ be the probability distribution. From this distribution, we can derive two sub probability distributions, one for the foreground (class A) and the other for the background (class B) given by $P^A = \{p_i\}_{i=1}^{t}$ and $P^B = \{p_i\}_{i=t+1}^{n}$ respectively respectively; where t is the threshold value. Subsequently the priori Tsallis entropy of generalized distributions for each distribution can be defined as follows,

$$H_q^A(t) = \frac{1}{q-1}\left[1 - \frac{\sum_{k=1}^{t} p_k^q}{\sum_{k=1}^{t} p_k}\right] \quad (7)$$

$$H_q^B(t) = \frac{1}{q-1}\left[1 - \frac{\sum_{k=t+1}^{n} p_k^q}{\sum_{k=t+1}^{n} p_k}\right] \quad (8)$$

3.2.2. Entropy fuzzification

In 1965, Zadeh introduced the conception of Fuzzy sets as an extension of the classical notion of a set [20]. Fuzzy sets are sets whose elements have degrees of membership. Mathematically, a fuzzy set, A is defined as set whose elements characterized by a one-to-one function called member function, $\mu_A(x_i)$ where x_i refers to the i-th element in the set. This membership function assigns a membership value to every element in the fuzzy set, which is suggestive of the amount of vagueness in the fuzzy set. The membership value of an element in a fuzzy set lies in [0,1]. A higher membership value refers to stark containment of the element in the set, while a lower value indicates weak containment.

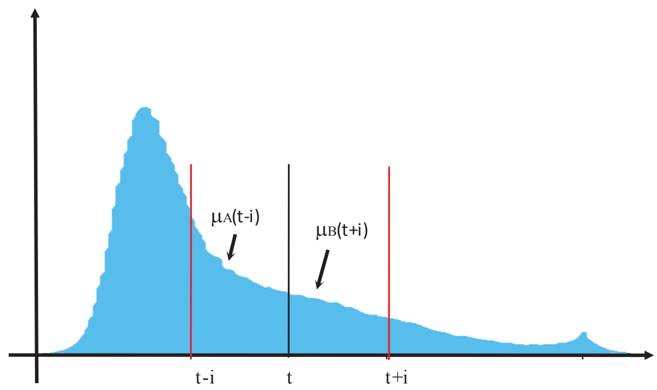

Figure 1: Fuzzy membership as an indication of how strongly a pixel belongs to its region.

The fuzzification of entropy at this juncture comprises the process of incorporating the fuzzy membership into the relations of entropy described by the equations (7) and (8). Hence, fuzzy segmentation deems the fuzzy memberships as an indication of how strongly a pixel value belongs to the background or to the foreground. Really, the farther away a value of pixel is from a presumed threshold (the deeper in its region), the greater becomes its probability to belong to a specific class. As a result, for any foreground and background pixel, which is i levels below or i levels above a given threshold t, the membership values are given by

$$\mu_A(t-i) = 0.5 + \frac{\sum_{k=0}^{i} p(t-k)}{2p(t)} \quad (9)$$

$$\mu_B(t-i) = 0.5 + \frac{\sum_{k=1}^{i} p(t+k)}{2(1-p(t))} \quad (10)$$

The above formulas give a measure of belonging a pixel to the foreground (class A) and background (class B) respectively, as shown in Fig. 1.

Evidently on the value corresponding to the threshold, one should have the maximum ambiguity, such that $\mu_A(t) = \mu_B(t) = 0.5$. Now, considering the two equations (9) and (10), the fuzzy form of entropic equations (7) and (8) can be written as

$$H_q^A(t) = \frac{1}{q-1}\left[1 - \frac{\sum_{k=1}^{t} \mu_A(k)^q}{\sum_{k=1}^{t} \mu_A(k)}\right] \quad (11)$$

$$H_q^B(t) = \frac{1}{q-1}\left[1 - \frac{\sum_{k=t+1}^{n} \mu_B(k)^q}{\sum_{k=t+1}^{n} \mu_B(k)}\right] \quad (12)$$

3.3. Obtaining the optimum threshold

In image processing, thresholding is the most regularly used method to distinguish objects from background. In

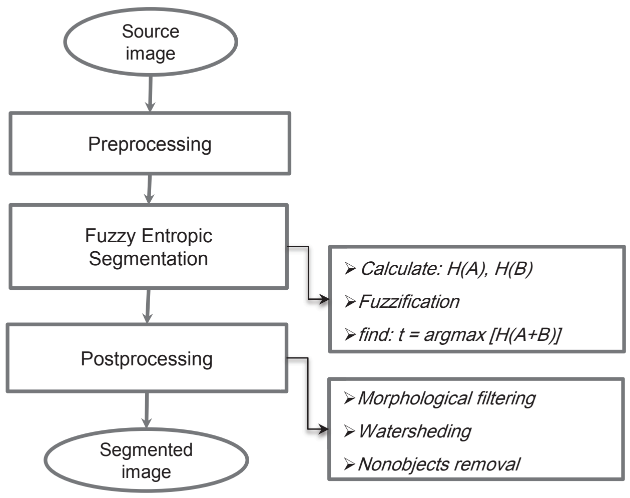

Figure 2: Block diagram of the proposed segmentation method.

this step the optimum threshold value t^* is automatically determined from maximizing the total entropy, $H_q^{(A+B)}(t)$. This value will be used for preliminary segmentation (thresholding). When total entropy is maximized, the value of parameter t that maximizes the function is believed to be the optimum threshold value [21]. Mathematically, the problem can be formulated as follows,

$$t^* = argmax[H_q^{A+B}(t)] = argmax[H_q^A(t) + H_q^B(t) + (1-q) \cdot H_q^A(t) \cdot H_q^B(t)] \tag{13}$$

In the case of RGB color images, the preceding scalar equation is replaced with the following vector equation:

$$\vec{t^*} = argmax[H_q^{A+B}(\vec{t})] = argmax[H_q^A(\vec{t}) + H_q^B(\vec{t}) + (1-q) \cdot H_q^A(\vec{t}) \cdot H_q^B(\vec{t})] \tag{14}$$

where $\vec{t} = (t_R, t_G, t_B)$ and the optimum threshold vector satisfies the identity

$$\|\vec{t^*}\| = \sqrt{(\omega_R t_R)^2 + (\omega_G t_G)^2 + (\omega_B t_B)^2} \tag{15}$$

where ω_R, ω_G and ω_B are the normalized energies of the channels R,G,and B respectively, i.e.,

$$\omega_R + \omega_G + \omega_B = 1 \tag{16}$$

3.4. Post-processing

This step consists of the following sub steps:

3.4.1. Morphological filtering

The ultimate goal of the Morphological filtering is to enhance the segmentation results obtained from the previous thresholding step. Due to the inconsistency within the color of objects, the binary image maybe contains some holes inside. The process of filling holes attempts to get rid of the holes form the binary image. This problem can be overcome by the filling holes process. Opening with small structure element is used to separate some objects that are still connected in small number of pixels [22, 23]. In image processing, dilation, erosion, filling holes and opening are all identified as morphological operations.

3.4.2. Watersheding

For this step, the well-known watershed algorithm [24, 25] is applied on the morphologically smoothed images to get finer final segmented results. Actually, the watershed algorithm is applied on the Euclidean Distance Transform (EDT) of the image. The EDT of a binary image works as: for each pixel in the binary image, the transform assigns a number that is the distance between that pixel and the nearest nonzero pixel of the image. The distance is calculated using the Euclidean distance metric. The peaks of the distance transform lay in the middle of each object. The idea is to run watershed using these peaks as markers. For this, we invert the distance transform so that the peaks become the regional minima the objects are correctly separated by watershed.

3.4.3. Wrong objects removals

This step contributes to remove incorrect objects according to the range of size of the object. Consequently tiny noise objects of sizes that are less than the minimum predefined threshold can be discarded. Also objects whose size greater than the maximum threshold size can be removed as well. It is worth to say that those thresholds are user-defined data and dependent on the application. The preceding steps of the proposed fuzzy entropic segmentation method can be depicted by the block diagram in Figure 2.

4. Experimental Results

In this section, results from a number of experiments are presented to allow comparison between the segmentation performance of the new method based on fuzzy Tsallis entropy and those of other competitive ones [18]. Firstly, to investigate the proposed approach for image segmentation, we have initiated by different image histograms. Each of these histograms describes the "foreground" and the "background". As illustrated earlier, the segmentation procedure searches for a luminance value that separates the two regions "foreground" and "background" in the image. This process allows judging the quality of segmentation result as function of some parameters such as amplitude, position and width of the peaks in the histograms. All these parameters have a key role in image characterization, such as homogeneity of the scene illumination (graylevel contrast), image and object size, "foreground" and "background" texture, noisy images, etc. To evaluate the performance of the proposed method, a variety of both synthetic and real images (approx.112 images whose graylevels are

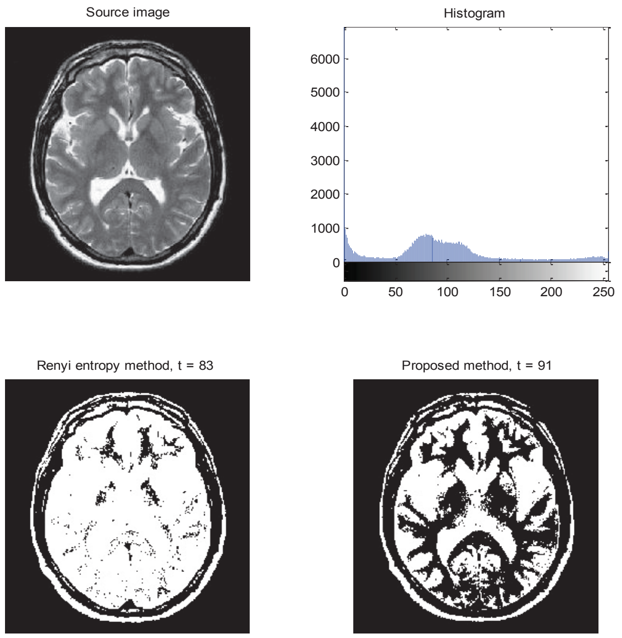

Figure 3: Segmentation results for an MR brain image produced by our method and by Renyi-entropy method, q = 0.8.

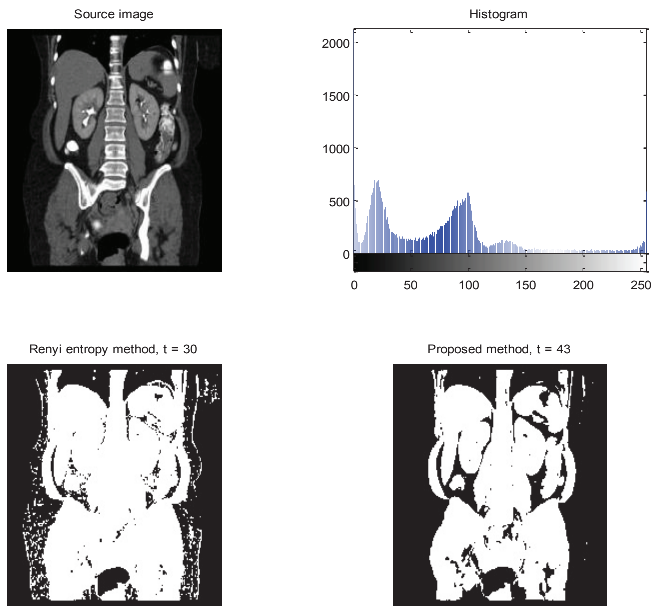

Figure 4: Segmentation results for a CT scan image produced by our method and by Renyi-entropy method, q = 0.8.

all quantized to 256 values) are used and several values of the parameter q are experimented. The results presented here were obtained at the different values of q parameter (q = 0.7, q = 0.8, and q = 0.9). In Fig. 3, an MR brain image with a heterogeneous distribution of light around it, leading to an irregular histogram of two peaks. It can be seen in this Figure that our method yields consistently more accurate segmentation results than the method reported in [26]. The proposed entropic method is shown by extensive experiments to be practical and of good robustness in such situations.

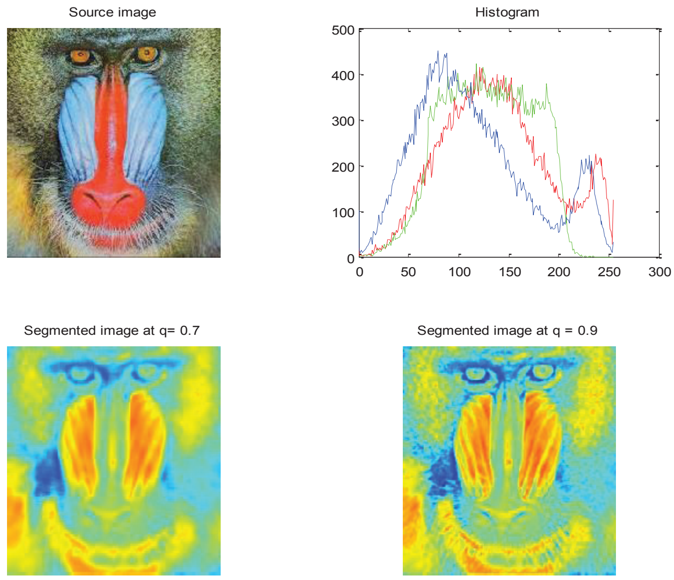

Figure 5: Original Baboon image and the segmentation results of applying our fuzzy entopic scheme.

Figure 4 provide more segmentation results to compare our method with the Renyi-entropy method [26]. In the figure, a CT (Computed Tomography) scan image of the abdomen and pelvisscan and the segmentation results generated by our method and Renyi-entropy method are shown. Furthermore, the proposed segmentation method is also applied on several standard images and the results of these implementations are extracted. For this purpose, commonly used images, i.e., baboon (see Figure 5), barbara (see Figure 6) and Lena (see Figure 7) are selected and the performance of applying the method on them is provided.

Figure 5 provides the segmentation results of Baboon image after applying the proposed segmentation scheme. In Figure 6, a "Barbara" image with a background of skin-like color around it, leading to a similarity between foreground color and background color. It is clear that the proposed fuzzy entropic approach can appropriately identify most regions of the image, especially when the approach takes into account the color information. Figure 7 shows a "Lena" image with a complicated background. In the figure, several regions on the face, hair and hat are interlaced. However, the proposed method is able to successfully segment the image into desired regions. The regions after segmentation are more consistent and are not affected by the inextricable background.

As a general remark, it is pertinent to mention here that most of the entropic segmentation methods proposed previously in literature are limited to segmenting of grayscale

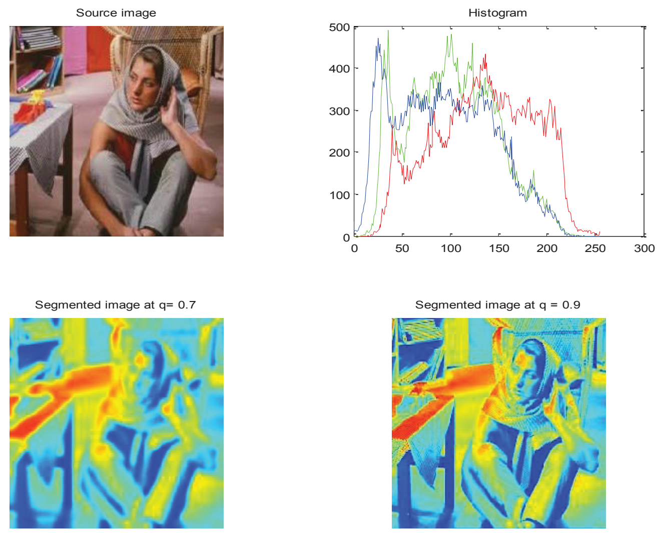

Figure 6: Original Barbara image and the segmentation results of applying our fuzzy entopic scheme.

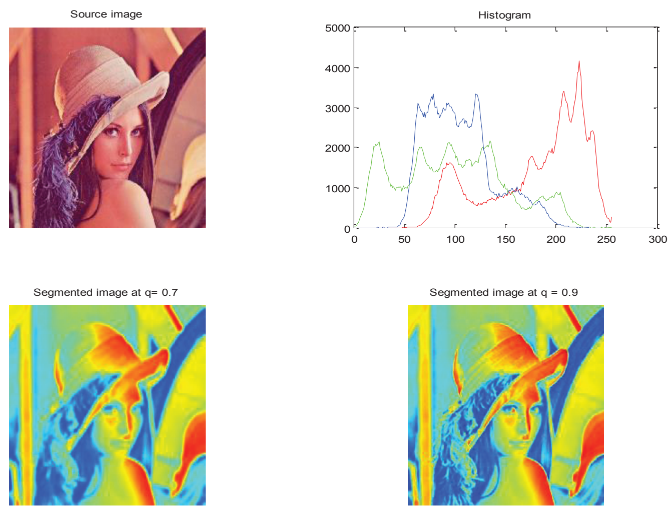

Figure 7: Original Lena image and the segmentation results of applying our fuzzy entopic scheme.

images in the spatial domain. On the other hand, the proposed method has been adapted and extended the concept of fuzzy entropy to be viable for segmenting color images. Furthermore, the method performs the task of segmentation rapidly and precisely. With respect to the execution time, on average, 2.4 seconds are required for segmenting an image of size 480×320 using an ordinary Desktop PC (Intel Core 2 Duo 2.4 GHz, 4 GB RAM) running Microsoft Windows 7 32-bit).

5. Conclusions

In this paper, an entropic method for image segmentation based on fuzzy Tsallis entropy has been developed and described. The method has been adapted for both grayscale and color image segmentation. Furthermore, it performs reasonably when applied to noisy images and/or images of complicated backgrounds, compared to other entropic segmentation approaches. The preliminary results have shown that the formalism of fuzzy entropy is more viable than the traditional entropy for segmentation purposes. An additional benefit of this method might come from its rapidity and easiness of implementation. Although the proposed method has been successfully applied to still images, it can be straightforwardly adapted to be applicable for image sequences due to its rapidity. In terms of future work, several experiments will be carried out on different standard images and some criteria such as number of the pixons in image, pixon to pixel ratio, normalized variance and computational time will be used to evaluate the method numerically.

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