Optimal BER in MIMO Raleigh Fading Channel from QPSK Modulation: Modified MMSE Versus ML Equalizer Evaluation

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Abstract

This paper proposes an exclusive equalizer alternative to existing optimum maximum likelihood (ML) equalizer that reduces bit error rate (BER) via spatial multiplexing, diversity combining and splits up each user's multiple input multiple output (MIMO) channel into parallel sub-channels. Quadrature phase shift keying (QPSK) modulation is treated here for the simulation purpose. The new equalizer is the modified version of minimum mean square error (MMSE) equalizer termed as Modified MMSE that incorporates channel equalization and noise addition with successive interference cancellation by means of modified matrix order. The process is same as SIC-Sort equalizer but matrix is modified by ordered optimum power of the transmitted symbol. Rayleigh fading channel is used in the simulation as it enables the computation of the expected complexity up to the linear term in the exponent. Simulations are done by MatLab that shows BER vs. signal-noise ratio (SNR) curve of Modified MMSE equalizer exceeds that of ML equalizer.

Keywords: ML, Modified MMSE, MIMO, QPSK, BER, SNR, Rayleigh fading channel.

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I. INTRODUCTION

MOST recently, MIMO is one of the most attractive techniques in wireless communication that uses multiple antennas at both transmitter and receiver and provides improved BER, or data rate compared to conventional communication systems [1-3]. MIMO has eminent features which offer significant increment in data throughput and link range without additional bandwidth and increase transmit power. Modern wireless communication standards such as IEEE 802.11n (Wi-Fi), 4G, 3GPP Long Term Evolution (LTE) and WiMAX has become an important part due to these properties [4-5]. In MIMO wireless communication, multipath fading is a usual phenomenon that causes ISI in the transmitted signal. To remove ISI from transmitted signal, BER reduction is compulsory. The ML equalizer would select the set of symbols that are closest in Euclidean distance to the received signals. In [6] the performance of ML detection has been analyzed over flat fading channels for wireless MIMO system and very high data rate can be obtained with little SNR penalty i.e. it has least possible BER but the transceiver structure will be complex exponentially with increasing transmitter size and modulation order. In [7] a soft-decoding ML MIMO demodulation method that lowers the complexity of the existing ML method has been proposed that offers optimum performance with significantly higher complexity than the linear equalizer. MMSE equalizer is a linear equalizer that minimizes mean square error (MSE) to maximize the post-detection signal-to-interference plus

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noise ratio (SINR). Still, it requires an accurate estimate of the amount of noise present in the system which is hard to obtain in practical systems. Performance analysis of the minimum-mean-square error (MMSE) linear multiuser detector is considered in an environment of non orthogonal signalling and additive white Gaussian noise. In [8], performance of the MMSE linear multiuser detector is considered in an environment of non orthogonal signalling and additive white Gaussian noise. A low-complexity equalizer which uses a combination of space-frequency MMSE filter and a pre-whitened maximum likelihood detector (MLD) is proposed in [9] for discrete Fourier transform precoded orthogonal frequency division multiple accesses (DFT-precoded-OFDM) systems employing multi-stream spatial multiplexing (SM). MMSE equalizer based receiver for MIMO wireless channel has been suggested in [10] which is a good choice for removing some ISI and minimizes the total noise power. Although it has lower complexity but resulting ISI from the transmitted signal degrades its performance which is yet a major drawback for MIMO wireless channel. For 3X3 MIMO communication systems, analysis of full-rate linear and space-time block code under a Rayleigh flat-fading environment has been carried out in [11] by using linear MMSE and ML receivers to minimize the average symbol error rate (SER) for a QPSK transmitted signal. For higher order MIMO the performance of ML equalizer is better than MMSE equalizer at the expense of compound hardware structure. By using spatial multiplexing technique a comparative study of various modulation schemes for MIMO wireless communication has been carried out [12]. Using channel estimation techniques, the BER performance characteristics of MIMO system has been investigated [13]. However non-linear detectors with SIC are more complex than linear detectors. Kuldeep et al. [14] proposed a different detection scheme for a 4X4 MIMO system and BER performance characteristics of ZF, MMSE and ML equalizers for MIMO wireless receiver has been investigated in [15]. However with QPSK modulation the existing optimum equalizer cannot exhibit excessive data rate in MIMO wireless communication. Selection of a novel equalizer that performs even better than existing equalizer is yet a major challenge in achieving optimal BER i.e. efficient modulation for MIMO wireless communication. Hence, in this research, due to the increasing demand of MIMO wireless communication, an innovative equalizer to achieve optimal BER performance with increased signal-noise-ratio (SNR) is strongly motivated.

II. MIMO CONFIGURATION

A MIMO system containing S transmit-antennas and T receive-antennas is shown in fig. 1.

![Figure 1: MIMO propagation system](image)

Where, S = 1, 2, 3...S, h_{ST} is the fading corresponding to the path from transmit antenna S to receive antenna T. n_t is the noise corresponding to receive antenna T. The received signal in the T^{th} antenna is given by

\[ y_t = \sum_{s=1}^{S} h_{ST} x_s + n_T \]  

(1)

III. MMSE EQUALIZER

MMSE equalizer is a more balanced linear equalizer that does not eliminate ISI entirely but minimizes total noise power and ISI components in the output. In wireless communications, MMSE equalizer approach minimizes the mean square error (MSE), which is a common measure of estimator quality. Let X is an unknown random variable, and let Y is a known random variable. An estimator \( \hat{X} \) is any function of the measurement Y, and its MSE is given by

\[ MSE = E[(\hat{X} - X)^2] \]  

(2)

Where, the expectation is taken over both X and Y. When it is not possible to determine a closed form for the MMSE equalizer then minimize the MSE within a particular class, such as the class of linear equalizers. The linear MMSE equalizer is the equalizer achieving minimum MSE among all the equalizers of the form \( AX + B \). Where, Y is a random vector, A is a matrix and b is a vector. Assuming the case where two symbols are interfered with each other. In the first time slot, the received signal on the first receive antenna is,

\[ y_1 = h_{1,1} x_1 + h_{1,2} x_2 + n_1 = [h_{1,1}, h_{1,2}] \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + n_1 \]

\[ y_2 = h_{2,1} x_1 + h_{2,2} x_2 + n_2 = [h_{2,1}, h_{2,2}] \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + n_2 \]

In matrix form, the above equation can be expressed as:
The above wireless channel is modulated by the theorem. The MMSE approach tries to find a coefficient $W$ which minimizes the criterion,

$$ E\left[|W y - x|^2 \right] $$

To solve we need to find a matrix $W$ which satisfies $WH = I$. The MMSE equalizer for satisfying this constraint is given by,

$$ W = \left[H^H H + N_0 I\right]^{-1} H^H $$

Where, $W$- equalization matrix and $H$- channel matrix. This matrix is known as the pseudo inverse for a general $S \times T$ matrix.

MMSE-SIC proceeds in the same way as MMSE. But in MMSE-SIC, noise term is added before the inversion step of equalization matrix. Using the MMSE equalization approach, the receiver can obtain an estimate of the two transmitted symbols $x_1$ and $x_2$ i.e.

$$ \begin{bmatrix} \hat{x}_1 \\ \hat{x}_2 \end{bmatrix} = \left[H^H H + N_0 I\right]^{-1} H^H \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} $$

Taking one of the estimated symbols for example $\hat{x}_2$ and subtract its effect from the received vector $y_2$, and $y_1$, i.e.

$$ \begin{bmatrix} r_1 \\ r_2 \end{bmatrix} = \begin{bmatrix} y_1 - h_{1,2} \hat{x}_2 \\ y_2 - h_{2,2} \hat{x}_2 \end{bmatrix} = \begin{bmatrix} h_{1,1} x_1 + n_1 \\ h_{2,1} x_1 + n_2 \end{bmatrix} $$

Expressing in matrix notation,

$$ \begin{bmatrix} r_1 \\ r_2 \end{bmatrix} = \begin{bmatrix} h_{1,1} \\ h_{2,1} \end{bmatrix} x_1 + \begin{bmatrix} n_1 \\ n_2 \end{bmatrix} $$

$$ r = hx_1 + n $$

### IV. MODIFIED MMSE EQUALIZER

In traditional SIC-Sort, the receivers remove successive interference by optimal power of the transmitted symbol and subtract its effect from the received symbol. As it has to consider power criterion for each of the transmitted symbol so its performance cannot exceed the ML equalizer. However, in novel MMSE, successive interference is cancelled based on modified matrix order, which is termed as modified MMSE. The process is same as SIC-Sort equalizer but matrix is ordered by optimum power of the transmitted symbol. This method offers even the greater performance compared to ML equalizer. In general, the performance of the linear MMSE detection method is worse than that of other nonlinear equalizer techniques. However, linear detection methods require a low complexity of hardware implementation. We can improve their performance without increasing the complexity significantly by modified MMSE method. It is a bank of linear equalizers, each of which detects one of the parallel data streams, with the detected signal components successively cancelled from the received signal at each stage. More specifically, the detected signal in each stage is subtracted from the received signal so that the remaining signal with the reduced interference can be used in the subsequent stage. Let $x_i$ denote the symbol to be detected in the $i$-th order, which may be different from the transmit signal at the $i$-th antenna, since $x_i$ depends on the order of detection. Let $\hat{x}_1$ denote a portion of $x_1$. For modified MMSE, MMSE method in equation (6) can be used for symbol estimation. Suppose that the MMSE method is used in the following discussion. The 1st stream is estimated with the 1st row vector of the MMSE weight matrix in equation (6). After estimation and segmentation to produce $\hat{x}_1$, the remaining signal in the first stage is formed by subtracting it from the received signal, that is,

$$ \hat{y}_1 = y - h_1 \hat{x}_1 = h_{1}(x_1 - \hat{x}_1) + h_{2}x_2 + \cdots + h_{c}x_c + z $$

If $x_1 = \hat{x}_1$, then the interference is successfully canceled in the course of estimating $x_2$.

### V. ML EQUALIZER

Maximum-Likelihood (ML) equalizer is a non linear equalizer in which a search is performed over all possible symbols and the most likely one is chosen. It calculates the Euclidean distance between the received signal vector and the product of all possible transmitted signal vectors with the given channel $H$, and finds the one with the minimum distance. No noise enhancement takes place and numerical issues are virtually not present, as no matrix inversions or divisions are necessary. The ML equalizer tries to search the most likelihood one which minimizes,

$$ J = \|y - Hx\|^2 $$

$$ J = \left\| \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} - \begin{bmatrix} h_{1,1} & h_{1,2} \\ h_{2,1} & h_{2,2} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \right\|^2 $$

In case of QPSK modulation the possible values are $4^2=16$. In order to find the maximum likelihood one, we have to find the minimum from the all sixteen combinations of

$$ l_{1111} = \left\| \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} - \begin{bmatrix} h_{1,1} & h_{1,2} \\ h_{2,1} & h_{2,2} \end{bmatrix} \begin{bmatrix} 11 \end{bmatrix} \right\|^2 $$
The estimate of the transmit symbol is chosen based on the minimum value from the above sixteen values. ML equalizer compares the received signals with all possible transmitted signal that is modified by channel matrix \( H \) and estimates transmit symbol vector \( x \) according to the Maximum Likelihood principle, which is shown as:

\[
x = \arg\max_{\mathbf{x}} p(y_1, \ldots, y_M | \mathbf{x}) = \arg\max_{\mathbf{x}} \prod_{i=1}^{M} p(y_i | \mathbf{x})
\]

where, \( x \) is the estimated symbol vector.

Although ML detection offers optimal error performance, it suffers from complexity issues. It has exponential complexity in the sense that the receiver has to consider \( |A|^3 \) possible symbols for \( S \) transmit antenna system with \( A \) is the modulation order.

### VI. QPSK MODULATION

QPSK is widely used in wireless communication that uses four points on the constellation diagram, equispaced around a circle. With four phases, QPSK can encode two bits per symbol, shown in the Fig. 2 with gray coding to minimize the BER. By quadrature means the signal shifts between phase states which are separated by 90 degrees. The signal shifts in increments of 90 degrees from 45 to 135, –45, or –135 degrees. These points are chosen as they can be easily implemented using an I/Q modulator. Only two I values and two Q values are needed and this gives two bits per symbol. There are four states because \( 2^2 = 4 \). As it has four states so it is a more bandwidth-efficient type of modulation than BPSK, possibly twice as efficient.

![QPSK constellation diagram](image)

#### V. Rayleigh Fading Channel

When there are many objects in the environment that scatter the radio signal, Rayleigh fading is an acceptable model before it arrives at the receiver. The central limit theorem expresses that, for sufficiently much scatter case, the channel impulse response will be well-modelled as a Gaussian process irrespective of the distribution of the individual components. If there is no dominant component to the scatter, then such a process will have zero mean and phase evenly distributed between 0 and 2\( \pi \) radians. The envelope of the channel response will therefore be Rayleigh distributed.

\[
p_Q(q) = \frac{2q}{\Omega} e^{-q^2}, q \geq 0 \tag{10}
\]

where, \( \Omega = E(Q^2) \)

The gain and phase elements of a channel's distortion are conveniently represented as a complex number. In this case, Rayleigh fading is exhibited by the assumption that the real and imaginary parts of the response are modelled by independent and identically...
VIII. SIMULATIONS AND PERFORMANCE ANALYSIS

The BER values have been computed as a function of SNR for QPSK modulation and different combinations of MIMO systems using MMSE, MMSE-SIC, Modified MMSE, and ML equalizers. The simulations studies have been carried out using MATLAB software.

Performance of modified MMSE equalizer over the existing optimum ML equalizer has been described by the figure 3, 4 and 5. QPSK modulation and 2x2, 2x3 and 2x4 MIMO configurations are applied for the performance analysis. From all of the three graphical analyses, it is evident that BER tends to decrease dramatically for modified MMSE equalizer compared to ML, MMSE and MMSE-SIC equalizers.

IX. CONCLUSIONS

In this paper, an idea about the optimal BER in MIMO wireless communication system at higher modulation levels and for different antenna configurations is presented. The performance are analysed for 2x2, 2x3 and 2x4 MIMO configurations in the forms of QPSK modulation under Rayleigh fading channel. A novel equalizer which is termed as modified MMSE that outperforms even ML equalizer in terms of data rate and BER performance is the outcome the present paper. While both equalizers are non-linear but ML equalizer tends to be exponential with higher transmit antennas and higher order modulation. The complexity of the proposed modified MMSE equalizer reduces exponential to linear giving optimal performance at the same time. The proposed modified MMSE method achieves optimal BER performance without requiring a large no. of signals.

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